

MANGALORE UNIVERSITY
DEPARTMENT OF MATHEMATICS
MSC MATHEMATICS

MTS 405	Number Theory	4 Credits (48 hours)
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Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Course Outcome: Students will have the knowledge and skills to explain the fundamental concepts and development of Elementary Number Theory using axioms, definitions, examples, theorems and their proofs.

Course Specific Outcome: At the end of the course students will have the knowledge and skills to:

- Apply the method of solving Linear Diophantine equations, Primality testing and factorization.
- Find the Dirichlet product of arithmetical functions, Dirichlet inverses.
- Solve the Linear congruences, Polynomial congruences modulo p , Simultaneous linear congruences, Simultaneous non-linear congruences, and Solving congruences modulo prime powers.
- Apply the properties of Legendre's symbol, Gauss lemma,
- Understanding the Pythagorean triples and their classification, Fermat's Last Theorem.
- Solving Pell's equation by continued fractions.

Unit I

Divisibility and Primes: Recapitulation of Division algorithm, Euclid's algorithm, Least Common Multiples, **Linear Diophantine equations**. Prime numbers and Prime-power factorisations, Distribution of primes, Fermat and Mersenne primes, **Primality testing and factorization**.

Arithmetical Functions: The Mobius function and its properties, Euler function, examples and properties, The **Dirichlet product of arithmetical functions, Dirichlet inverses** and the Mobius inversion formula.

(12 Hours)

Unit II - Congruences:

Recapitulation of basic properties of congruences, Residue classes and complete residue systems, **Linear congruences**. Reduced residue systems and the Euler-Fermat theorem, **Polynomial congruences modulo p** and Langrange's theorem, Simultaneous linear congruences, **Simultaneous non-linear congruences**, An extension of Chinese Remainder Theorem, **Solving congruences modulo prime powers**.

(12 Hours)

Unit III - Quadratic Residues and Quadratic Reciprocity Law:

Quadratic residues, **Legendre's symbol and its properties**, Euler's criterion, Gauss lemma, The quadratic reciprocity law and its applications, The Jacobi symbol, Applications to Diophantine equations.

(12 Hours)

Unit IV - Sums of squares, Fermat's last theorem and Continued fractions:

Sums of two squares, Sums of four squares, **The Pythagoras theorem, Pythagorean triples and their classification**, Fermat's Last Theorem (Case $n = 4$).

Recapitulation of **Finite continued fractions**, Infinite continued fractions, Representation of irrational numbers, Periodic continued fractions and quadratic irrationals. **Solution of Pell's equation by continued fractions**.

(12 Hours)

References

- [1] G. A. Jones and J. M. Jones, *Elementary Number Theory*, Springer UTM, 2007.
- [2] Tom M. Apostol, *Introduction to Analytic Number Theory*, Springer, 1989.
- [3] David M. Burton, *Elementary Number Theory*, 7th Ed., McGraw-Hill, 2010.
- [4] Niven, H.S. Zuckerman & H.L. Montgomery, *Introduction to the Theory of Numbers*, Wiley, 2000.
- [5] H. Davenport, *The Higher Arithmetic*, Cambridge University Press, 2008.