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BSCMTC 382

Choice Based Credit System VI Semester B.Sc. Degree

Examination, September 2022

(CBCS) (2021-22 Batch Onwards)

MATHEMATICS

Linear Algebra – VIII (a)

Time : 3 Hours

Max. Marks : 80

- Instructions :** 1) Answer **any ten** questions from Part – **A**. **Each** question carries **2** marks.
2) Answers to Part – **A** should be written in the **first few** pages of the answer book **before** answers to Part – **B**.
3) Answer **any twelve** questions from Part – **B**. **Each** question carries **5** marks.
4) Scientific calculators are **allowed**.

PART – A

I. Answer **any 10** questions :

(10×2=20)

- 1) If $T : V \rightarrow V'$ is a linear transformation, then prove that $\text{Ker } T$ is a subspace of V .
- 2) Let $T : V \rightarrow V$ be a linear transformation of V onto V , then prove that T is $(1, 1)$ mapping if and only if T is onto.
- 3) Let $V = \mathbb{R}^2 = V'$ and let $T : V \rightarrow V'$ be a linear transformation which is rotated through θ° about the origin with respect to the basis $\{(1, 0), (0, 1)\}$ of V and V' , find the matrix of linear transformation.
- 4) Show that the ring $M_n(F)$ has zero divisors.
- 5) Find the row rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 2 & 3 \\ -3 & -6 & -9 & -3 \end{bmatrix}$.
- 6) Define the rank of a linear transformation.

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- 7) If A and B are similar matrices, then prove that $\text{rank } A = \text{rank } B$.
- 8) If $A, B \in M_n(F)$, then show that, $\text{rank } (A + B) = \text{rank } A + \text{rank } B$ by giving an example.
- 9) If $\sum_{j=1}^n a_{ij}x_j = 0, i = 1, 2, \dots, m$ be a system of 'm' homogeneous linear equations in 'n' unknowns. If $m < n$, prove that the system always has a nontrivial solution.
- 10) Find the minimal polynomial of the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$.
- 11) Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.
- 12) Show that the characteristic roots of $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ are the fourth roots of unity.
- 13) If $A \in M_n(F)$ and A^T be the transpose of A , then prove that $P_A(t) = P_{A^T}(t)$.
- 14) If $A \in M_n(F)$ is a nilpotent matrix, then prove that all its characteristic roots are zeros.

PART – B

II. Answer **any 12** questions :

(12×5=60)

- 1) Prove that a linear transformation $T : V \rightarrow V'$ is (1, 1) mapping if and only if $\text{Ker } T = \{0\}$.
- 2) Let $T : V \rightarrow V'$ be a linear transformation of V onto V' , with $\text{Ker } T = W$. Prove that $\frac{V}{W} \cong V'$.
- 3) Show that a linear transformation $T : V \rightarrow V'$ is (1, 1) if and only if it maps linearly independent set in V onto linearly independent set in V' .



- 4) Let $V = \mathbb{C}$, the space of complex numbers and $V' = \mathbb{R}^2$. Prove that the mapping $T : V \rightarrow V'$ defined by $T(a + ib) = (a, b)$ is a linear transformation. Also prove that T is an isomorphism.
- 5) Show that if A is nilpotent matrix then $(I + A)$ is non-singular.
- 6) Let $V = \mathbb{R}^3$ and let $T : V \rightarrow V'$ be a linear transformation defined by $T(x, y, z) = (x', y', z')$ where $x' = 2x$, $y' = 4y$, $z' = 5z$. Find the matrix of T with respect to the basis $\left(\frac{2}{3}, 0, 0\right), \left(0, \frac{1}{2}, 0\right), \left(0, 0, \frac{1}{4}\right)$ of V .
- 7) Let $V = V' = \mathbb{R}^3$ and let $T : V \rightarrow V'$ be a linear transformation defined by $T(x, y, z) = (x', y', z')$ where $x' = x$, $y' = 2y$, $z' = x + y + z$. Find the rank of T .
- 8) Using elementary row operations, find the row rank of the matrix A ,
- $$A = \begin{bmatrix} 6 & -2 & -18 \\ -4 & 1 & 11 \\ -5 & 2 & 16 \end{bmatrix}.$$
- 9) Find A^{-1} if $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ by performing elementary row operations.
- 10) Show that the system of equations has only trivial solution
- $$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ 2x_1 + 3x_2 + x_3 &= 0 \\ 3x_1 + 6x_2 + 5x_3 &= 0. \end{aligned}$$
- 11) Prove that a system of n , non homogeneous equations in n unknowns has a unique solution if and only if the associate matrix is non singular.
- 12) Find the dimension of the space of solutions if
- $$\begin{aligned} 2x_1 + x_2 - 3x_3 - x_4 &= 0 \\ x_1 + 2x_3 &= 0 \\ 3x_1 - x_2 - x_3 + x_4 &= 0. \end{aligned}$$



- 13) Let $A \in M_n(F)$ and let $q(x) \in F[x]$ be a minimum polynomial of A . If $f(x) \in F[x]$ is any other polynomial satisfied by A , then prove that $q(x)$ divides $f(x)$.
- 14) Let $A \in M_n(F)$ with $q(x) = a_0 + a_1x + a_2x^2 + \dots + x^m$ be the minimum polynomial of A . Then prove that A is non singular if and only if $a_0 \neq 0$.
- 15) Find the minimum polynomial of the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -9 \\ 0 & 1 & 6 \end{bmatrix}$.
- 16) Let $A \in M_n(F)$ and $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_m \in F$ be distinct characteristic roots of A . If $v_1, v_2, v_3, \dots, v_m$ are the corresponding characteristic vectors, then prove that $v_1, v_2, v_3, \dots, v_m$ are linearly independent.
- 17) Find the characteristic roots of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.
- 18) If A and $B \in M_n(F)$ are such that B is similar to A , then prove that $P_A(t) = P_B(t)$.
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