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MTH 454

II Semester M.Sc. Degree Examination, September/October 2022
(CBCS – New Syllabus)
MATHEMATICS
Topology

Time : 3 Hours

Max. Marks : 70

- Note :** 1) Answer **any five full** questions.
2) **No** additional sheets will be **provided** for answering.
3) **Use** of scientific calculator is **permitted**.

1. a) Show that $\tau = \{\mathbb{R}\} \cup \{G \subseteq \mathbb{R} : 0 \notin G\}$ is a topology on the set $\mathbb{R}$ of all real numbers.
b) If there exists a homeomorphism of a topological space X onto a metric space Y , then show that X is metrizable.
c) Define closure $\bar{A}$ of a set A in a topological space X . Show that $\bar{A}$ is the smallest closed set containing the set A . If $\mathbb{N}$ denotes the set of all positive integers, then find $\bar{\mathbb{N}}$ in the space $(\mathbb{R}, \tau)$, where τ is given in the problem 1 (a). **(3+6+5)**
2. a) Let A be a subset of a topological space X . Prove that a point $x \in \bar{A}$ if and only if each neighborhood of x intersects A .
b) Let $f : X \rightarrow Y$ be a mapping of one topological space into another. Show that the following statements are equivalent :
i) $f^{-1}(F)$ is closed in X whenever F is closed in Y .
ii) $f(\bar{A}) \subseteq \overline{f(A)}$ for every subset A of X .
c) Prove or disprove: If X is a non-empty set, Y is a topological space, and $f : X \rightarrow Y$ is any map, then $\{f^{-1}(V) : V \text{ is open in } Y\}$ is a topology on X . **(5+6+3)**
3. a) If S is a class of subsets of a non-empty set X , show that S can serve as an open subbase for a topology on X .
b) Show that $\overline{X - A} = X - \text{Int}(A)$ for every subset A of a topological space.
c) Prove or disprove : Every second countable space is first countable. **(7+4+3)**

P.T.O.



4. a) Define the weak topology generated by a class of functions. Consider a function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} 0 & \text{if } x \leq 1 \\ 2 & \text{if } x > 1 \end{cases} \forall x \in \mathbb{R}$. Give a complete description of the weak topology on $\mathbb{R}$ generated by the function f .
- b) Let f be a mapping of a topological space X into a product space $\prod_{i \in I} X_i$. Show that f is continuous if and only if $\pi_i \circ f : X \rightarrow X_i$ is continuous for each projection π_i .
- c) Prove that a topological space is compact if and only if every class of closed sets with the finite intersection property has non-empty intersection. **(4+5+5)**
5. a) Show that every closed and bounded interval $[a, b]$, $a, b \in \mathbb{R}$, of the real line $\mathbb{R}$ is compact.
- b) State and prove Tychonoff's theorem.
- c) Prove or disprove : Every compact subspace of a topological space is closed. **(6+6+2)**
6. a) Prove that a topological space X is a T_1 -space if and only if every finite set is closed in X .
- b) Let $\{X_i : i \in I\}$ be a non-empty class of topological spaces. Prove that the product space $\prod_{i \in I} X_i$ is Hausdorff if and only if X_i is Hausdorff for each $i \in I$.
- c) Prove or disprove : A finite T_1 -space is discrete. **(6+6+2)**
7. a) Prove that every compact Hausdorff space is normal.
- b) Let X be a first countable space, Y be a topological space, $f : X \rightarrow Y$ be a map. Prove that the following statements are equivalent :
- $f : X \rightarrow Y$ is continuous.
 - For every sequence $x_n \rightarrow x$ in X , $f(x_n) \rightarrow f(x)$ in Y .
- c) State Urysohn's Lemma. **(6+6+2)**
8. a) Show that a topological space X is disconnected if and only if there exists a continuous mapping of X onto the discrete two-point space $\{0, 1\}$.
- b) Prove that the product of any non-empty class of connected spaces is connected. **(5+9)**
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