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MTH 552

IV Semester M.Sc. Degree Examination, Sept./Oct. 2022
(CBCS – New Syllabus)
MATHEMATICS
Complex Analysis – II

Time : 3 Hours

Max. Marks : 70

Notes : 1) Answer **any five full** questions.

2) **No** additional sheets will be provided for answering.

3) **Use** of scientific calculator is **permitted**.

4) **All** questions carry **equal** marks.

1. a) If $f(z)$ is meromorphic in Ω with the zeros a_j and the poles b_k , then show that

$$\frac{1}{2\pi i} \int_{\gamma} \frac{f'(z)}{f(z)} dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, b_k) \text{ for every cycle } \gamma \text{ which is homologous}$$

to zero in Ω and does not pass through any of the zeros or poles.

- b) How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2 ?

- c) Find the poles and residues of the following function

i) $\frac{1}{\sin^2 z}$

ii) $\frac{1}{z^m(1-z^n)}$

where $m, n \in \mathbb{Z}_+$.

(7+3+4)

2. a) Suppose that $f(z)$ is analytic in the annulus $r_1 < |z| < r_2$ and continuous on the closed annulus. If $M(r)$ denote the maximum of $|f(z)|$ for $|z| = r$, show that $M(r) \leq M(r_1)^\alpha M(r_2)^{1-\alpha}$ where $\alpha = \log(r_2/r) : \log(r_2/r_1)$.

- b) Derive Poisson's formula for harmonic function.

(7+7)

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3. a) If the function $f_n(z)$ are analytic and $\neq 0$ in a region Ω and if $f_n(z)$ converges to $f(z)$, uniformly on every compact subset of Ω , then show that $f(z)$ is either identically zero or never equal to zero.

- b) Show that the series

$\zeta(z) = \sum_{n=1}^{\infty} n^{-s}$ converges for $\text{Re } z > 1$, and represents its derivatives in series form.

- c) What is the co-efficient of z^7 in the Taylor development of $\tan z$? Also develop $\log (\sin z/z)$ in powers of z upto the term z^5 . (6+3+(3+2))

4. a) Show that $\frac{\pi^2}{\sin^2 \pi z} = \sum_{n=-\infty}^{+\infty} \frac{1}{(z-n)^2} + g(z)$ where $g(z)$ is analytic in the whole complex plane.

- b) Prove that

$$\sin \pi(z + \alpha) = \sin(\pi\alpha) e^{\pi z \cot \pi\alpha} \prod_{n=-\infty}^{+\infty} \left(1 + \frac{z}{n + \alpha}\right) e^{-z/(n+\alpha)}. \quad (7+7)$$

5. a) Obtain Jensen's formula.

- b) Using Poisson's formula derive Poisson-Jensen's formula. (10+4)

6. a) Show that there exists a basis (w_1, w_2) such that the ratio $r = w_2/w_1$ satisfies the following conditions :

i) $\text{Im } \gamma > 0$

ii) $-\frac{1}{2} < \text{Re } \gamma \leq \frac{1}{2}$

iii) $|\gamma| \geq 1$

iv) $\text{Re } \gamma \geq 0$ if $|\gamma| = 1$.

Further show that the ratio γ is uniquely determined by these conditions and there is a choice of two, four, or six corresponding bases.

- b) Show that an elliptic function without poles is a constant function. (10+4)



7. a) Prove that $\prod_{n=1}^{\infty} \left(1 + \frac{z}{n}\right) e^{-z/n}$ converges absolutely and uniformly on every compact subset of $\mathbb{C}$.

b) Show that the function $\theta(z) = \prod_{n=1}^{\infty} (1 + h^{2n-1} e^z) (1 + h^{2n-1} e^{-z})$ where $|h| < 1$ is analytic in the whole plane and satisfies the functional equation $\theta(z + z \log h) = h^{-1} e^{-z} \theta(z)$. **(6+8)**

8. a) Show that any even elliptic function for which '0' is neither a zero nor a pole can be written as :

$$C \prod_{n=1}^n \frac{P(z) - P(a_k)}{P(z) - P(b_k)} .$$

b) Show that $P(z) - P(u) = \frac{-\sigma(z-u)\sigma(z+u)}{(\sigma+z)^2(\sigma(u))^2}$. **(10+4)**
