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STH 452

II Semester M.Sc. Degree Examination, September/October 2022
STATISTICS
Distribution Theory

Time : 3 Hours

Max. Marks : 70

Note : Question No. 1 is **compulsory**. Answer **any four** questions from the remaining seven questions.

1. Answer **any six** sub-divisions. **Each** question carries **3** marks. **(6×3=18)**

- Find the m.g.f. of normal random variable with parameters μ and σ^2 .
- If X and Y have joint pdf $f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$. Examine whether X and Y are independent.
- Define non-central t distribution. State its mean and variance.
- Prove that, $V(X) = V(E(X|Y)) + E(V(X|Y))$.
- Let $X \sim U(0, 1)$. Find the transformation $Y = g(X)$ which has the density function $f_Y(y) = e^{-y}$, $y > 0$.
- Prove or disprove : Exponential distribution satisfies lack-of-memory property.
- Define Wishart distribution and mention any one of its properties.
- Let $Y_1, Y_2, \dots, Y_n$ are order statistics of a random sample from an exponential distribution with parameter λ . Find the joint distribution of Y_1, Y_n .

Answer **any four full** questions from the following, **each** question carries

13 marks.

(4×13=52)

2. a) Obtain p.g.f. of a negative binomial distribution, show that negative binomial is a members of power series distribution.

b) State and establish any two properties of Weibull distribution.

(7+6)

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3. a) If X is any continuous random variable with cdf F . Find the distribution of $Y = F(X)$.
 b) Define truncated distribution. Write down the probability mass function of a Poisson distribution truncated at zero and find its mean and variance.
 c) What is probability integral transformation ? What is its use ? **(5+5+3)**
 4. a) Let X and Y be iid gamma random variables. Find the joint distribution of $U = \frac{X}{X+Y}$ and $V = (X + Y)$. Obtain the conditional distribution of V given $U = u$.
 b) Show that a random variable X has standard Cauchy distribution if and only if $\frac{1}{X}$ also has standard Cauchy distribution.
 c) Define extreme value distribution and state its important properties. **(6+4+3)**
 5. a) Prove or disprove : Mean and variance of a random sample of a normal distribution are independent.
 b) Define logistic distribution. Obtain its moment generating function. **(8+5)**
 6. a) Let $Y = (Y_1, Y_2, \dots, Y_n)'$ where Y_i 's are iid $N(0, 1)$ random variables. Prove that a necessary and sufficient condition that $Y'AY$ has chi-square distribution is that A is idempotent.
 b) Let $X_1, X_2, \dots, X_n$ be a random sample from exponential with parameter θ . Let $Y_1, Y_2, \dots, Y_n$ be order statistics. Then show that $Y_s - Y_r$ and Y_r are independent. **(5+8)**
 7. a) Derive the characteristic function of Wishart distribution.
 b) Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Show that $\sum_{i=1}^n (X_i - \bar{x})^2 / \sigma^2$ has χ^2 -distribution with $n-2$ degrees of freedom. **(7+6)**
 8. a) Define multivariate normal distribution. Find the distribution of linear combinations of components of a vector having multivariate normal distribution.
 b) Prove or disprove : The conditional distribution of any sub-vector given the other components in a multivariate normal random vector is again multivariate normal. **(7+6)**
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