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STH 453

II Semester M.Sc. Examination, September/October 2022

STATISTICS

Theory of Point Estimation

Time : 3 Hours

Max. Marks : 70

Note : Question No. 1 is **compulsory**. Answer **any four** questions from the remaining seven questions. Figures to **right** indicate marks to sub-questions.

Answer **any 6** of the following :

(6×3=18)

1. a) Demonstrate that $B(1, 2/2)$ is not complete.
b) State the likelihood principle of obtaining minimal sufficient statistics.
c) Given a sample of size n from $N(\theta, 1)$, obtain Fisher information about θ .
d) Based on a random sample of size n from $\text{Exp}(\lambda)$, obtain Moment estimator of $\lambda > 0$.
e) Prove that UMVU estimator is unique.
f) Define consistent estimator and state invariance property of consistent estimator.
g) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from Poisson distribution with parameter θ . Show that sample mean is CAN estimator for θ .
h) Given a random sample from $U(0, \theta)$ obtain MLE of θ .
2. a) Define Fisher information for one and several parameter models. Obtain the Fisher information contained in a sample of size n from Cauchy distribution with median θ .
b) Explain the concept of completeness. Is the Bernoulli family complete ? Justify your answer. **(7+6)**
3. a) Obtain a sufficient statistics for θ of $U(0, \theta)$ distribution and examine whether it is complete.
b) State the factorization theorem and prove it in the discrete case. **(6+7)**

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4. a) State and Prove Rao-Blackwell-Lehman-Scheffe theorem.
 b) Given a random sample of size n from Poisson distribution with parameter λ . Let N be the number of observations which are equal to zero. Obtain unbiased estimator for $e^{-\lambda}$ and improve the estimator using Rao-Blackwell Theorem. (5+8)

 5. a) State and Prove Cramer-Rao inequality. Use it obtain the UMVUE for θ based on a random sample of size n from exponential distribution with mean θ .
 b) State and Prove necessary and sufficient conditions for an estimator to be UMVUE. (8+5)

 6. a) State and prove invariance property of ML estimator.
 b) Under the regularity conditions prove that MLE is a consistent estimator for θ . (7+6)

 7. a) Given a random sample of size n from a lognormal distribution with parameters μ and σ , obtain moment estimators for μ and σ .
 b) Given a set of iid r.v's $X_1, X_2, \dots, X_n$ from an exponential distribution with location parameter θ and scale parameter σ . Obtain MLE's of (μ, σ) . (6+7)

 8. a) Given a random sample of size n from a Gamma $(\alpha, 1)$ distribution, obtain MLE's of $(\alpha, 1)$. Show that the estimators is CAN.
 b) Given a set of iid r.v's $X_1, X_2, \dots, X_n$ from a Binomial (n, p) distribution, where both n and p are unknown, obtain moment estimators of (n, p) . Prove or disprove that moment estimators are jointly consistent for (n, p) . (5+8)
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