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**BSCMTEN 201**

**II Semester Open Elective (NEP 2020) Examination, September 2022
(2021 – 2022 Batch Onwards)
MATHEMATICS – II (Science Stream)**

Time : 2 Hours

Max. Marks : 60

- Instructions :** 1) Answer **any eight** questions from Part – **A**. **Each** question carries **3** marks.
 2) Answers to Part – **A** should be written in the **first few** pages of the answer book before answers to Part – **B**.
 3) Answer **any six** questions from Part – **B** by choosing **two** questions from **each** Unit. **Each** question carries **6** marks.
 4) Scientific calculators are **allowed**.

PART – A

Answer **any eight** questions.**(8×3=24)**

1. Define a binary operation on the set of positive integers by $a * b = \max \{a, b\}$. Show that the operation is both associative and commutative.
2. On the set of integers $\mathbb{Z}$, $*$ is defined by $a + b - 1 \forall a, b \in \mathbb{Z}$. Find the inverse of 2.
3. Form the table for multiplication modulo 10 for the set $\{1, 3, 7, 9\}$.
4. Construct the Klein's four group.
5. Evaluate : $\lim_{(x,y) \rightarrow (1,1)} \frac{xy - y - 2x + 2}{x - 1}, x \neq 1$.
6. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x, y) = x^2 + 3xy + y - 1$.
7. Find $\frac{dw}{dt}$ for $w = x^2y - y^2$ with $x = \sin t$ and $y = e^t$.
8. Prove that $f(x, y) = \frac{\sqrt{x} + \sqrt{y}}{x + y}$ is a homogeneous function of degree $-\frac{1}{2}$.

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9. Evaluate $\int_C (x + y) ds$, where C is the straight line segment $x = t$, $y = 1 - t$, $z = 0$ from $(0, 1, 0)$ to $(1, 0, 0)$.
10. Evaluate $\int_0^3 \int_0^2 (4 - y^2) dy dx$.
11. Sketch the region of integration $\int_0^\pi \int_0^x (x \sin y) dy dx$.
12. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dx dy dz$.

PART – B

Answer **any six** questions by choosing **two** questions from **each** Unit. **(6×6=36)**

Unit – I

13. On the set of all integers Z , define a binary operation $*$ by $a * b = a + b + 1$, $a, b \in R$. Prove that $(Z, *)$ is an abelian group.
14. Let $G = Q - \{-1\}$ be the set of all rational numbers except -1 and $*$ be a binary operation on G defined by $a * b = a + b + ab \forall a, b \in G$. Prove that $(G, *)$ is an infinite abelian group.
15. Prove that the set $G = \{3, 6, 9, 12\}$ forms a finite abelian group of order 4 under multiplication modulo 15.
16. Find the orders of the elements of the additive modulo 6 group $(Z_6, +_6)$.

Unit – II

17. Show that the function $f(x, y) = \frac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$ using two path test.
18. Show that $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies Laplace equation.
19. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$ if exists.
20. Verify Euler's theorem for the function $f(x, y) = x^2 + 2hxy + y^2$.



Unit – III

21. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1).
22. Sketch the region of integration of the double integral $\int_0^1 \int_2^{4-2x} dy dx$ and write an equivalent double integral with the order of integration reversed and evaluate.
23. Find the area bounded by $y = x$ and $y = x^2$ in the first quadrant.
24. Evaluate :
- a) $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz dy dx$
- b) $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx .$
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